the integrals taken over the waveguide cross section.

Following the work of Kornhauser,[3] a trial function can be used to obtain an approximate value for the eigenvalue for the fundamental mode.[4]

Dropping subscripts, a legitimate trial function for the fundamental mode is

$$\psi(x) = \sum_{n=0}^{N} C_{2n+1}x^{2n+1}. \quad (3)$$

Note that the trial function does not satisfy the boundary condition, also that the parameters C_{2n+1} will be adjusted to minimize the eigenvalue.

Let $N=1$; then

$$\psi = C_1x + C_3x^3. \quad (4)$$

By inserting the trial function in (2), the eigenvalue takes the following form:

$$k^2 = \frac{\alpha C_1^2 + \beta C_1C_3 + \gamma C_3^2}{\delta C_1^2 + \epsilon C_1C_3 + \zeta C_3^2}, \quad (5)$$

where $\alpha, \beta, \gamma, \delta, \epsilon$ and ζ are constants dependent on the dimensions of the waveguide.

The eigenvalue k^2 is next minimized with respect to C_1 and C_3. After eliminating C_1 and C_3, k^2 is obtained from

$$k^4 - 2\Theta k^2 + \Phi = 0, \quad (6)$$

where

$$\Theta = \frac{2\alpha\zeta + 2\delta\gamma - \epsilon\beta}{4\delta\zeta - \epsilon^2}$$

$$\Phi = \frac{4\alpha\gamma - \beta^2}{4\delta\zeta - \epsilon^2}.$$

For the waveguide of semicircular side walls and flat top and bottom walls, the coefficients are

$$\alpha = a\left[p + \frac{\pi}{4}r\right]$$

$$\beta = a^3\left[2p^3 + \frac{3\pi}{2}p^2r + 4pr^2 + \frac{3\pi}{8}r^3\right]$$

$$\gamma = a^5\left[\frac{9}{5}p^5 + \frac{9\pi}{4}p^4r + 12p^3r^2 + \frac{27\pi}{8}p^2r^3 + \frac{24}{5}pr^4 + \frac{9\pi}{32}r^5\right]$$

$$\delta = a^3\left[\frac{1}{3}p^3 + \frac{\pi}{4}p^2r + \frac{2}{3}pr^2 + \frac{\pi}{16}r^3\right]$$

$$\epsilon = a^5\left[\frac{2}{5}p^5 + \frac{\pi}{2}p^4r + \frac{8}{3}p^3r^2 + \frac{3\pi}{4}p^2r^3 + \frac{16}{15}pr^4 + \frac{\pi}{16}r^5\right]$$

$$\zeta = a^7\left[\frac{1}{7}p^7 + \frac{\pi}{4}p^6r + 2p^5r^2 + \frac{15\pi}{16}p^4r^3 + \frac{8}{3}p^3r^4 + \frac{15\pi}{32}p^2r^5 + \frac{48}{105}pr^6 + \frac{5\pi}{256}r^7\right],$$

where $r=b/a$ and $p=1-r$.

[3] E. T. Kornhauser and I. Stakgold, "Application of Variational Methods to the Equation $\nabla^2u+\lambda u=0$," Cruft Lab., Harvard Univ., Cambridge, Mass., Tech Rept. No. 117; 1950.

[4] The fundamental mode is the equivalent to the H_{10} mode in rectangular waveguide and the H_{11} mode in circular waveguide

The numerical results, the experimental points and the points obtained from Montgomery, Dicke and Purcell[2] are shown in Fig. 3.

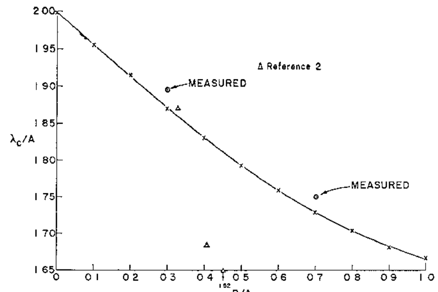

Fig. 3—The cutoff wavelengths for the waveguide with flat tops and bottoms and semicircular waveguides as a function of aspect ratio.

The method has also been applied to calculate the cutoff wavelengths for truncated-circular waveguides.[5]

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[5] G. R. Valenzuela, "The Cut-Off Wavelengths and Power-Voltage Impedance for Composite Waveguides for the Fundamental Mode," Rad. Lab., The Johns Hopkins Univ., Baltimore, Md., Tech. Rept., to be published.

A Mechanical Waveguide Hybrid Phase Shifter*

Some interesting results have been obtained on a waveguide-hybrid phase shifter under development for use in an S-band radio-astronomy interferometer antenna.

The phase shifter[1,2] illustrated in Fig. 1(a) comprises a waveguide hybrid junction, a rotating chopper, and a pair of variable lengths of a short-circuited waveguide: the variable lengths of the guide are provided by a movable short-circuit plunger. The operation of the phase shifter is as follows.

Let us assume that the chopper is initially closed. When power is injected at port 1, the power enters the hybrid junction where it is divided, one half being coupled to the adjacent waveguide where it continues to propagate in the same direction, while the other half of the power propagates in the main waveguide. The properties of the hybrid junction are such that the signal in the adjacent waveguide is in phase quadrature, lagging the signal in the main waveguide. At the chopper the two signals are reflected; power in the main waveguide enters the hybrid junction, recombining in phase with the power in the adjacent arm. The entire power leaves port 2, provided that the hybrid is matched. If, now, the chopper is opened, an additional path length is introduced into the hybrid system and the amount of phase shift is equivalent to an electrical length of twice the stub length. The phase shift can be made to be proportional to either the stub length s or to $(\lambda_g/2)-s$, where λ_g is the guide wavelength, depending upon how the chopper is introduced across the waveguide.

A sketch of the chopper used in the model under investigation is shown in Fig. 1(b). In the radio-astronomy application the chopper rotates at a speed of 900 rpm. The phase-shift characteristics as a function of chopper rotation are shown in Fig. 2 for two different plunger positions. Note particularly that the shorter stub length provides the greater amount of phase shift. For this case the phase shift is proportional to $(\lambda_g/2)-s$. This result can be explained quite readily if one analyzes the behavior of the chopper and stub in terms of an equivalent circuit.

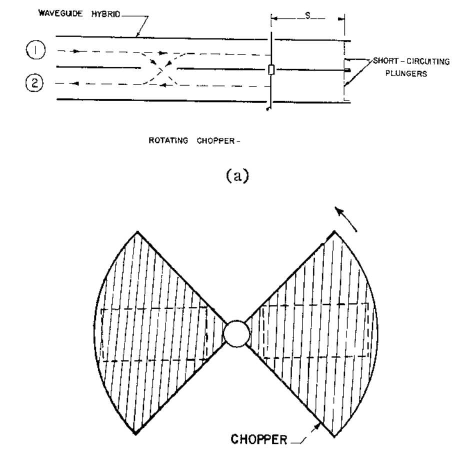

Fig. 1(a) Waveguide hybrid phase shifter. (b) Rotating chopper used in the phase shifter.

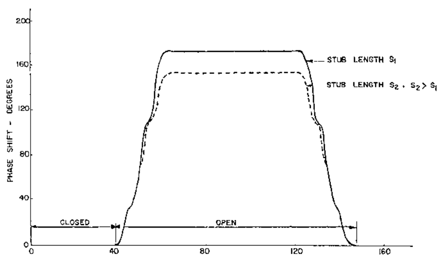

Fig. 2—Phase-shift characteristics as a function of chopper rotation.

The susceptance of the short-circuited stub is plotted as a point Y_{s1} on the Smith Chart diagram shown in Fig. 3 Placed in shunt with Y_{s1} is the susceptance of the chopper which behaves as a capacitive iris (positive susceptance). As the chopper is introduced, the net susceptance is reduced,

* Received by the PGMTT, April 17, 1961.

[1] H. Shnitkin, "Survey of electronically scanned antennas," *Microwave J.*, vol. 4, pp. 57–64; January, 1961.

[2] J. Blass, *et al.*, "A High Speed X-Band Conical Scanner," AF Cambridge Res. Ctr., Bedford, Mass., AFCRC Rept. TR-58-145-11; April, 1958.

causing the point to move counterclockwise around the Smith Chart. When the chopper is completely closed, the net susceptance becomes infinite, and this position is described by the point Y_{sc}. Hence the total phase shift is equivalent to twice the length of line $Y_{s1}-Y_{sc}$ or proportional to $(\lambda_g/2)-s$.

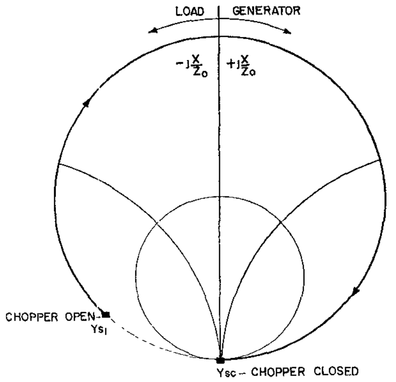

Fig. 3—Smith Chart representation illustrating the operation of the phase shifter.

It is not difficult to demonstrate that, if one provides a chopper which is equivalent to an inductive iris (negative susceptance), then the phase shift would be equivalent to twice the length of line $Y_{s1}-Y_{sc}$ measured in the opposite direction around the chart. In this case the amount of phase shift is proportional to the stub length s.

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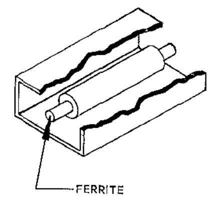

Fig. 1—Reggia-Spencer rectangular waveguide phase shifter geometry.

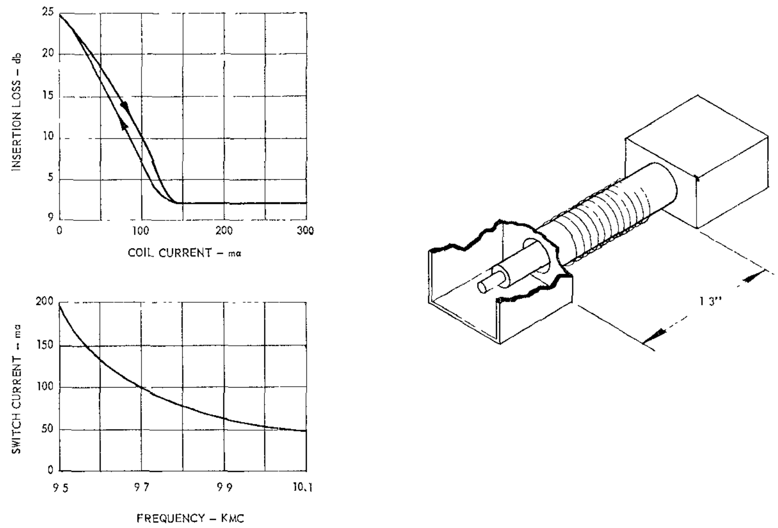

Fig. 2—Insertion loss vs coil current for reactive switch, normally off. Ferrite R-1, 0.260-inch diameter ×2.0 inches long. Cutoff guide section, 1.3 inches long ×0.4-inch diameter. Frequency is 9.6 kMc. Insert shows frequency sensitivity of switching current for minimum loss.

Rectangular Waveguide Switches*

Introduction

Since the first disclosure by Reggia and Spencer[1] of the rectangular waveguide phase shifter, a number of investigators have presented qualitative analyses describing observed performances.[2-4] In general, these authors agree on two phenomena that occur. First, upon application of a longitudinal magnetic field to the ferrite (phase-shifter geometry shown in Fig. 1), there is a "trapping" process whereby the RF field is increasingly concentrated in the ferrite acting as if it were a "dielectric" waveguide. Second, there is a mode-conversion process that takes place simultaneously (probably a circularly polarized wave in the ferrite rod) that gives rise to an electric-field component in the ferrite region orthogonal to the electric field in the empty waveguide (TE_{10} mode).

It is possible to utilize these two effects to design a number of different reactive and absorption switches. It is the purpose of this paper to relate these devices to the phase-shifter phenomena and thus classify them. It is also desired to present preliminary experimental and behavioral data.

Types of Switches

1) *Reactive Switch, Normally Off:* Fig. 2 shows the geometry of the reactive switch.[5] Its operation depends upon the "trapping effect" mentioned earlier. Since the small tube containing the ferrite is normally at "cutoff," there is no energy propagation. When the magnetic field is applied, the energy is trapped in the ferrite and propagates through the cutoff section with low insertion loss. The input and output waveguides can be either in-line or crossed. This switch is similar to the tetrahedral junction described by Weiss[6] and Caswell.[7]

2) *Reactive Switch, Normally On:* Fig. 3(a) shows a reactive switch which is normally on. It is constructed with a thin metal septum, normal to the electric field of the dominant mode, soldered to the waveguide narrow-wall and extending to the ferrite in close proximity. With the absence of an applied field, energy is transmitted with low insertion loss. When a field is applied, the "mode-conversion" process takes effect, and an orthogonal electric-field component is generated so that the energy is reflected. The performance of the reactive switch is shown in Fig. 3(b).

3) *Absorption Switch, Normally On:* The switch shown in Fig. 3 is easily modified into an absorption switch by replacing the thin metal septum with a thin sheet of resistive material. The absorption switch is shown in Fig. 4(a) and its performance is plotted in Fig. 4(b). This switch would normally have a low VSWR for all coil currents. Reggia[8] has recently suggested a switch of this type. This switch design splits the ferrite and sandwiches the lossy material between the ferrite halves. In the design of Fig. 4, a resistive material was mounted close to the ferrite.

* Received by the PGMTT, May 8, 1961.

[1] F. Reggia and E. G. Spencer, "A new technique in ferrite phase shifting for beam scanning of microwave antennas," Proc. IRE, vol. 45, pp. 1510–1517; November, 1957.

[2] P. A. Rizzi and B. Gatlin, "Rectangular guide ferrite phase shifters employing longitudinal magnetic fields," Proc. IRE., vol. 47, pp. 446–447; March, 1959.

[3] A. Clavin, "Reciprocal ferrite phase shifters in rectangular waveguide," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, p. 334; July, 1958.

[4] Jerold A. Weiss, "A phenomenological theory of the Reggia-Spencer phase shifter," Proc. IRE, vol. 47, pp. 1130–1137; June, 1959.

[5] A. Clavin, "Problems Associated with Rectangular Waveguide Phase Shifters," presented at PGMTT Natl. Symp., San Diego, Calif.; May 9–11, 1960.

[6] J. A. Weiss, "The tetrahedral junction as a waveguide switch," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 120–121; January, 1960.

[7] P. A. Caswell, "Performance of Tetrahedral Junction Waveguide Switches," presented at the 6th Symp. on Magnetism and Magnetic Material, New York, N. Y.; November, 14–17, 1960.

[8] F. Reggia, "A new broadband absorption modulator for rapid switching of microwave power," Internatl. Solid State Circuits Conf., University of Pennsylvania, Philadelphia, Pa.; February, 1961.